Chapter 4
Section 4.4

Solving Logarithmic Equations: We will use all of our rules of logarithms in order to solve equations involving a logarithm.
Ex: Solve $log(x) + log(x+2) = log(6x+1)$

$$\log(x(x+2)) = \log(6x+1)$$
$$x^2+2x = 6x+1$$

Grp Ex: Solve $a) log(x-3) = 4$,
$b) log(x) - log(x-1) = 2$ and
$c) 2 \cdot ln(x) = ln(x+3) + ln(x-1)$

a) $10^4 = x-3 \qquad 10{,}000 = x-3 \qquad 10{,}003 = x$

b) $\log\left(\frac{x}{x-1}\right) = 2 \qquad 10^2 = \frac{x}{x-1} \qquad 100x - 100 = x \qquad x = \frac{100}{99}$

c) $\ln(x^2) = \ln((x+3)(x-1)) \qquad x^2 = x^2 + 2x - 3 \qquad 2x = 3 \qquad x = \frac{3}{2}$

Solving Exponential Equations: We will also use the fact the logarithms and exponentials are inverses to solve exponential equations.
Ex: Find an exact answer for $6^x = 7^{x-1}$

$$\log(6^x) = \log(7^{x-1})$$
$$x\log(6) = (x-1)\log(7)$$
$$x(\log(7) - \log(6)) = \log(7)$$
$$x = \frac{\log(7)}{\log(7) - \log(6)}$$

Grp Ex: Find the exact solutions to $a) (1.02)^{4t-1} = 5$ and
$b) 3^{2x-1} = 5^x$

a) $\log_{1.02}(5) = 4t - 1 \qquad t = \frac{\log_{1.02}(5) + 1}{4}$

b) $\ln(3^{2x-1}) = \ln(5^x)$

$$(2x-1)\ln(3) = x\ln(5)$$
$$2x\ln(3) - x\ln(5) = \ln(3)$$
$$x(2\ln(3) - \ln(5)) = \ln(3)$$
$$\boxed{x = \frac{\ln(3)}{2\ln(3) - \ln(5)}}$$

Radioactive Dating: It has been found that the amount A of a radioactive substance remaining after t years is given by

$$A = A_0 e^{rt}$$

where A_0 is the initial amount present and r is the annual rate of decay. A standard measurement of the speed of decay is half-life. We can use this formula to determine the age of ancient rocks using a method known as potassium-argon dating.

Ex: There was a recent dinosaur find in Utah. Paleontologists want to estimate the age of the sauropods (type of dinosaur) by dating the volcanic debris in the surrounding rock using potassium-argon dating. The half-life of potassium-40 is 1.31 billion years. If 92.4% of the original amount of potassium-40 is still present in the rock, the how old is the rock?

To find r

$$\frac{1}{2}A_0 = A_0 e^{r(1.31\times 10^9)}$$

$$\frac{1}{2} = e^{r(1.31\times 10^9)}$$

$$-\ln(2) = (1.31\times 10^9)r$$

$$r = \frac{-\ln(2)}{1.31\times 10^9} \approx -5.29\times 10^{-10}$$

To find t

$$.924 = 1 \cdot e^{(-5.29\times 10^{-10})t}$$

$$\ln(.924) = (-5.29\times 10^{-10})t$$

$$t = \frac{\ln(.924)}{(-5.29\times 10^{-10})} \approx 150 \text{ million years,}$$

Newton's Model for Cooling: Newton found that when a cold object is surrounded by a hot object the difference between them decreases exponentially according to the formula

$$D = D_0 e^{kt}$$

where D_0 is the initial difference, k is a constant according to the objects and t is time.

Ex: A turkey with temperature of $40°F$ is moved to a $350°F$ oven. After 4 hours the internal temperature of the turkey is $170°F$. If the turkey is done when the temperature reaches $185°F$, then how much longer must it cook?

$$D_0 = 350 - 40 = 310$$

When $t = 4$, $D = 180 = 350 - 170$

Solve for k.

$$180 = 310 e^{k\cdot 4}$$

Practice: 5, 11, 21, 29, 40, 44, 48, 52, 56, 85

$$\frac{18\cancel{0}}{31\cancel{0}} = e^{4k}$$

$$\ln(18) - \ln(31) = 4k$$

$$k = \frac{\ln(18)-\ln(31)}{4} \approx -0.1359$$

Solve for t now

Turkey done when reaches 185°

When $D = 350 - 185 = 165$.

$$165 = 310 e^{-0.1359t}$$

$$-0.1359t = \ln(165/310)$$

$$t = \frac{\ln(165/310)}{-0.1359} \approx 4.6404$$